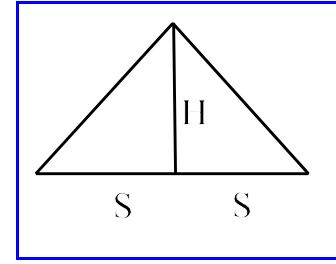


## Mat 123 Practice Questions

1. (15 points) A conical hole (figure 1) is to be 6 feet in **diameter** and 20 feet deep is to be dug in rock where the density of the rock is a function of the depth. If the weight density of the rock is  $\rho(y) = \sqrt{y} \frac{\text{lbs}}{\text{ft}^3}$ , where  $y$  is measured from the surface, determine the work done needed to dig the hole.



**Figure 1**

2. (15 points) A circular oil slick has density of oil  $\rho(r) = \frac{1}{1+r^2} \frac{\text{gms}}{\text{m}^2}$ . Find the total mass of oil in a slick of radius 2 m.

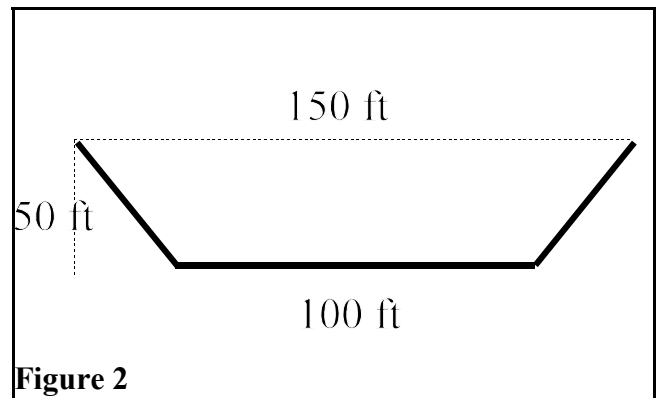
3. (20 points) A house  $L$  feet long and  $2S$  wide has crawl space attic of height  $H$

(Figure 2) that is filled insulation foam that has a density  $\rho(y) = \frac{1}{1+y^2} \frac{\text{gms}}{\text{ft}^3}$  as measured from the floor. Find an expression for the total mass of foam if the attic is filled with foam.

4. (15 points) A plate in the shape of a parabola  $y = 1 - 3x^2$  and bounded by the lines  $x \geq 0$  and  $y \geq 0$  has paint applied to the surface. The density of the paint varies with  $y$  and is given by  $\rho = y^3 \frac{\text{gms}}{\text{mm}^2}$ . Find an expression (integral) that determines the mass of paint on the plate.

- A. A block of ice weighing 100 lbs is being lifted by a chain that weighs 2 lbs/ft. If the ice is lifted at a rate of 4 ft/sec and is melting at a rate of 0.02 lbs/sec, find the work done in lifting the block with the chain a distance of 50 ft.

- B. A glass plate on the front of a tank (figure 2) holds back a liquid that has a density  $y^2$  lbs/ft<sup>2</sup>, where  $y$  is measured from the top. Find the hydrostatic pressure exerted on the plate.



**Figure 2**

- C. A *horn* is formed by revolving the graph  $y = \sqrt{\frac{1}{1+Lx}}$ ,

$0 \leq x \leq 2$  about the  $x$  axis. If the bowl is filled with a material with density  $\rho(x) = \frac{1}{1+Mx} \frac{\text{oz}}{\text{ft}^3}$ , find the total weight of material in the bowl of height 2.  $L$  and  $M$  are positive constants.

2. (15 points) In *Star Trek: The Penultimate Sequel*, A Klingon Warbird is hit with a photon torpedo which sends antimatter into space in a **spherical** pattern. If the density of the antimatter is given by  $\rho(r) = \frac{1}{\sqrt{r^8+1}}$ , determine if the amount of antimatter is finite or infinite. If finite, provide an estimate.

4. (10 points) Paint is applied to a plate given in figure 2. The density of paint varies from bottom to top according to the formula  $\rho(y) = \sqrt{1+y^2} \frac{\text{gm}}{\text{mm}^2}$ . Find the total mass of paint on the plate.

